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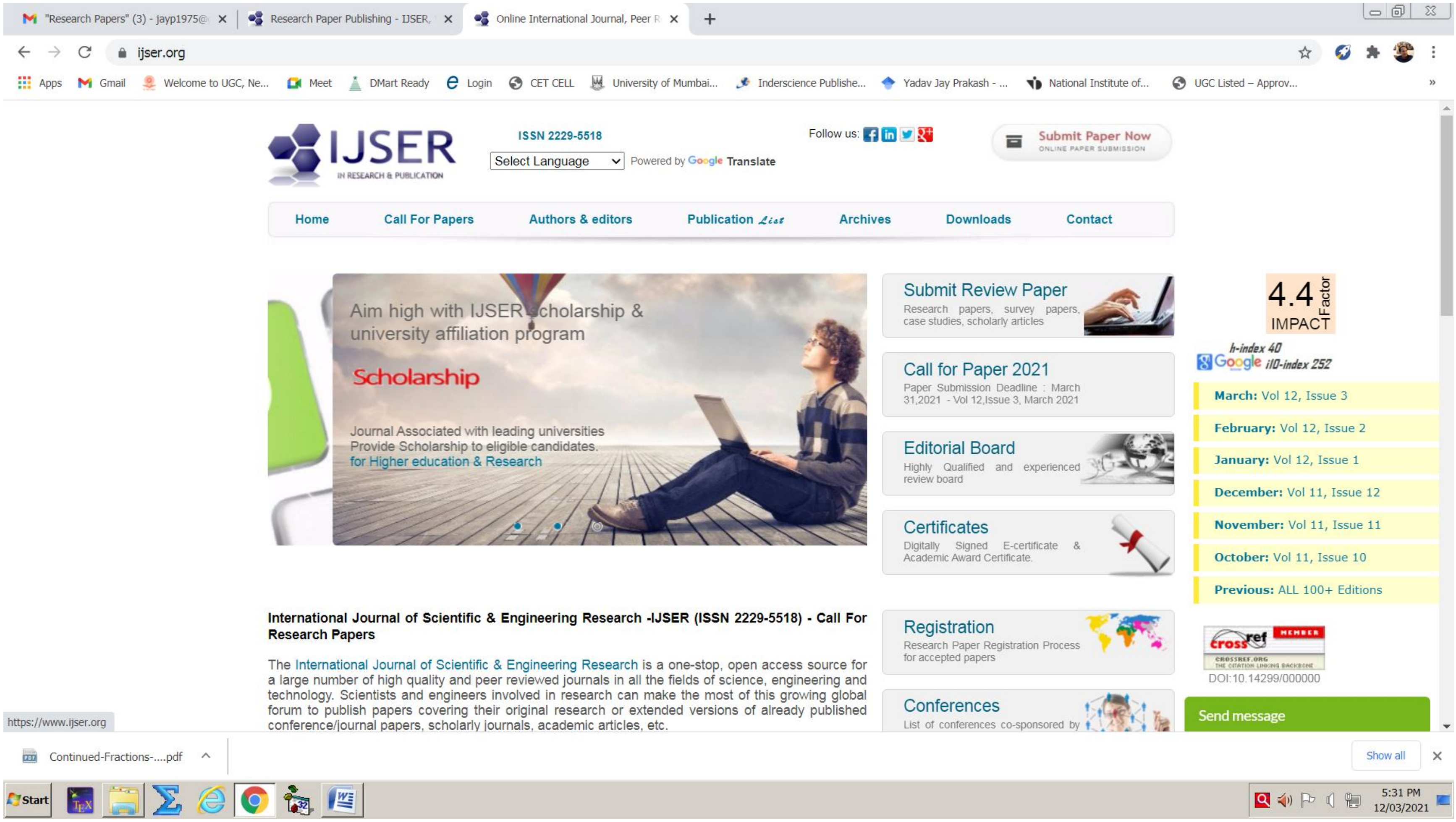
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Hyper-geometric Polynomial with Continued Fraction

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Abstract: Continued fractions, premeditated since the moment of antique Greece, only became an authoritative means in the 18th century, in the hands of the enormous mathematician. This paper enlightens how Euler established the proposal of hyper-geometric polynomials and pooled up the two domains. Continued fractions play an important role in number theory and classical analysis ever since the time of Euler and Gauss. The most fascinating function of this article is to discuss various outcomes and applications domain continued fraction in terms of hyper-geometric polynomials. As continued fractions turn out to be more significant in fraction suitable to their use in discovery of algorithms in **estimation theory**, illustration of the continuing consequence of their control.

Index Terms: Hyper-geometric polynomials, continued fraction, q- fractional operators, and Euler's theory.

I. CONTINUED FRACTION

Continued fraction expansions are implied in the Euclidian algorithm and are significant in giving coherent approximation of actual information. . Any pair $x_0 > x_1$ of positive integers generates a decreasing sequence $x_0 > x_1 > x_2 > \cdots$ in the set N of all positive integers:

$$\left.\begin{aligned} x_0 &= b_0x_1 + x_2, \\ x_1 &= b_1x_2 + x_3, \\ x_2 &= b_2x_3 + x_4, \\ &\vdots \\ x_{n-2} &= b_{n-2}x_{n-1} + x_n \\ x_{n-1} &= b_{n-1}x_n \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

With $b_j \in N$, $j = 0, 1, \ldots$ Since any decreasing sequence in N is finite, there exists $n \in N$ such that for $x_{n-1} = b_{n-1}x_n$ the algorithm stops at this line. This is the standard form of the Euclidean algorithm, which provides a foundation for multiplicative number theory.

We will consider above mentioned equation as a system of linear algebraic equations with integer coefficients $b_0, b_1, b_2, \ldots$. Eliminating the unknowns x_k from that equation, we will obtain

$$\frac{x_{k-1}}{x_k} = b_{k-1} + \frac{1}{x_k/x_{k+1}} \qquad k=1,2,\ldots\ldots \qquad \ldots\ldots (2)$$

Which obviously yields the development of x0/x1 into a finite regular continued fraction?

$$\left.\frac{x_0}{x_1} = b_0 + \cfrac{1}{b_1 + \cfrac{1}{b_2 + \cfrac{1}{\ddots + \cfrac{1}{bn}}}}\right\} \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

the continued fraction is written in line form:

$$\frac{x_0}{x_1} = b_0 + \frac{1}{b_1 +}\frac{1}{b_2 + \ldots\ldots +}\frac{1}{b_{n-1}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

This shows that any rational number equals the value of a regular continued fraction.

II. HYPER GEOMETRIC POLYNOMIALS

The Jacobi polynomials generally termed as hyper geometric polynomials $P_n^{(\alpha,\beta)}$(x) are a class of classical orthogonal polynomials. They are orthogonal with respect to the weight $(1-x)^{\alpha}$ $(1+ x)^{\beta}$ on the interval [−1, 1].

Through Hyper-Geometric Function

The hyper geometric polynomials are defined via the hyper geometric function as follows:

$$P_n^{(\alpha,\beta)}(z) = \frac{(\alpha+1)_n}{n!}\,{}_2F_1\left(-n, 1+\alpha+\beta+n; \alpha+1; \tfrac{1}{2}(1-z)\right) \qquad \ldots.. (5)$$

Where $(\alpha+1)_n$ is Pochhammer's symbol (for the rising factorial). In this case, the series for the hyper geometric function is finite; therefore one obtains the following equivalent expression:

$$P_n^{(\alpha,\beta)}(z) = \frac{\Gamma(\alpha+n+1)}{n!\,\Gamma(\alpha+\beta+n+1)} \sum_{m=0}^{n} \binom{n}{m} \frac{\Gamma(\alpha+\beta+n+m+1)}{\Gamma(\alpha+m+1)} \left(\frac{z-1}{2}\right)^m \qquad \ldots\ldots (6)$$

III. ESTIMATION THEROY

Approximation theory is a branch of mathematics, a quantitative part of functional analysis. Approximation theory is concerned with how functions can best be approximated with simpler functions, and with quantitatively characterizing the errors introduced thereby. Note that what is meant by best and simpler will depend on the application. Diophantine approximation deals with approximations of real numbers by rational numbers. Approximation usually occurs when an exact form or an exact numerical number is unknown or difficult to obtain. However some known form may exist and may be able to represent the real form so that no significant deviation can be found.

Now we move to the approximation of real numbers by rational numbers. Our aim is to use the lowest denominator rational number possible and still get a nice approximation. How good a rational approximation can one get to a given real number α? One trivial rational approximation to α is a number a/q with

$$\alpha - a/q \quad \leq 1/2q , \qquad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

ESTIMATION CONSTANT:

Let $x \in \Omega := [0, 1] \setminus \Phi$. The expansion of x as a regular continued fraction is denoted by

$$x = [O; a_l, a_2, a_3 \ldots.] \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

and the corresponding sequence of convergent by

$$\left(\frac{P_n}{q_n}\right) \quad -1 \leq n \qquad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The operator $T: \Omega \to \Omega$ is defined by

$$T_x := \frac{1}{x} - [-\frac{1}{x}] \qquad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Hence, if x has the expansion (1.1)

then $$T_x = [0; a_2, a_3, a_4 \ldots.] \qquad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Therefore T is called the one-sided shift operator connected with the regular continued fraction. Finally we introduce the sequence of approximation constants (θn), $-1 \leq n$ with

$$\theta n := q_n | q_n x - p_n |, -1 \leq n \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Each convergent can be expressed explicitly in terms of the continued fraction as the ratio of certain multivariate polynomials called *continuants*.

If successive convergents are found, with numerators h_1, h_2, ... and denominators k_1, k_2, ... then the relevant recursive relation is:

$$h_n = a_n h_{n-1} + h_{n-2},$$
$$k_n = a_n k_{n-1} + k_{n-2}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The successive convergents are given by the formula

$$h_n/k_n = a_n h_{n-1} + h_{n-2}/a_n k_{n-1} + k_{n-2}. \quad \ldots\ldots\ldots\ldots\ldots (14)$$

REPRESENTATION OF POLYNOMIALS:

Number	r	0	1	2	3	4	5	6	7	8	9	10
123	a_r	**123**										
	ra	123										
12.3	a_r	**12**	**3**	**3**								
	ra	12	37/3	123/10								
1.23	a_r	**1**	**4**	**2**	**1**	**7**						
	ra	1	5/4	11/9	16/13	123/100						
0.123	a_r	**0**	**8**	**7**	**1**	**2**	**5**					
	ra	0	1/8	7/57	8/65	23/187	123/1 000					
$\phi = \sqrt{5} + 1/2$	a_r	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	ra	1	2	3/2	5/3	8/5	13/8	21/13	34/21	55/34	89/55	144/89

−ϕ = −√5 + 1/2	***a_r***	**−2**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	ra	−2	−3/2	−5/3	−8/5	−13/8	−21/13	−34/21	−55/34	−89/55	−144/89	−233/144
√2	***a_r***	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**
	ra	1	3/2	7/5	17/12	41/29	99/70	239/169	577/408	1 393/985	3 363/2 378	8 119/5 741
1⁄√2	***a_r***	**0**	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**
	ra	0	1	2/3	5/7	12/17	29/41	70/99	169/239	408/577	985/1 393	2 378/3 363
√3	***a_r***	**1**	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**
	ra	1	2	5/3	7/4	19/11	26/15	71/41	97/56	265/153	362/209	989/571
1⁄√3	***a_r***	**0**	**1**	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**	**1**
	ra	0	1	1/2	3/5	4/7	11/19	15/26	41/71	56/97	153/265	209/362
√3⁄2	***a_r***	**0**	**1**	**6**	**2**	**6**	**2**	**6**	**2**	**6**	**2**	**6**
	ra	0	1	6/7	13/15	84/97	181/209	1 170/1 351	2 521/2 911	16 296/18 817	35 113/40 545	226 974/262 087
³√2	***a_r***	**1**	**3**	**1**	**5**	**1**	**1**	**4**	**1**	**1**	**8**	**1**
	ra	1	4/3	5/4	29/23	34/27	63/50	286/227	349/277	635/504	5 429/4 309	6 064/4 813
e	***a_r***	**2**	**1**	**2**	**1**	**1**	**4**	**1**	**1**	**6**	**1**	**1**
	ra	2	3	8/3	11/4	19/7	87/32	106/39	193/71	1 264/465	1 457/536	2 721/1 001
π	***a_r***	**3**	**7**	**15**	**1**	**292**	**1**	**1**	**1**	**2**	**1**	**3**
	ra	3	22/7	333/106	355/113	103 993/33 102	104 348/33 215	208 341/66 317	312 689/99 532	833 719/265 381	1 146 408/364 913	4 272 943/1 360 120
Number	***r***	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

Continued fractions have also been used in **modelling optimization** problems for **wireless network virtualization** to find a route between a source and a destination.

ra: rational approximant obtained by expanding continued fraction up to a_r

Find the continued fraction for $3.245 = \frac{649}{200}$

Step	Real Number	Integer part	Fractional part	Simplified	Reciprocal of f
1	$r = \frac{649}{200}$	$i = 3$	$f = \frac{649}{200} - 3$	$= \frac{49}{200}$	$\frac{1}{f} = \frac{200}{49}$
2	$r = \frac{200}{49}$	$i = 4$	$f = \frac{200}{49} - 4$	$= \frac{4}{49}$	$\frac{1}{f} = \frac{49}{4}$
3	$r = \frac{49}{4}$	$i = 12$	$f = \frac{49}{4} - 12$	$= \frac{1}{4}$	$\frac{1}{f} = \frac{4}{1}$
4	$r = 4$	$i = 4$	$f = 4 - 4$	$= 0$	STOP

Continued fraction form for $3.245 = \frac{649}{200} = [3; 4, 12, 4]$

$$= 3 + \cfrac{1}{4 + \cfrac{1}{12 + \cfrac{1}{4}}}$$

Mathematical Applications:

1. **Differential Equations:** The logarithmic derivatives of some hyper geometric functions for which quadratic transformations exist are solutions of Painlevé equations.
2. **Conformal Mappings:** The quotient of two solutions of maps the closed upper half-plane Iz≥ 0; conformally onto a curvilinear triangle. Hyper geometric functions, especially complete elliptic integrals, also play an important role in quasi conformal mapping.
3. **Group Representations:** For harmonic analysis it is more natural to represent hyper geometric functions as a Jacobi function. For special values of α and β there are many group-theoretic interpretations. First, as spherical functions on non compact Riemannian symmetric spaces of rank one, but also as associated spherical functions, intertwining functions, matrix elements of SL(2,R)(2,ℝ), and spherical functions on certain non symmetric Gelfand pairs. Harmonic analysis can be developed for the Jacobi transform either as a generalization of the Fourier-cosine transforms or as a specialization of a group Fourier transforms.
4. **Combinatory:** In combinatory, hyper geometric identities classify single sums of products of binomial coefficients.
5. **Monodrama Groups:** The three singular points in Riemann's differential equation lead to an interesting Riemann sheet structure. By considering, as a group, all analytic transformations of a basis of solutions under analytic continuation around all paths on the Riemann sheet.

CONCLUSION:

Continued fractions constitute a major branch of estimation theory because they have many applications within the field. First of all, they provide us with a method to find the best rational approximations of a finite and infinite numbers in the sense that no other rational with a smaller denominator is a better estimation. Continued fractions allow one to find solutions of polynomial equations with ease. Furthermore, continued fractions can be put to use in the rationalization of large integers.

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